Effect of Hub Motor Mass on Stability and Comfort of Electric Vehicles

D.J. van Schalkwyk and M.J. Kamper
Electrical Machines Laboratory, Department of Electrical and Electronic Engineering, University of Stellenbosch
Stellenbosch, South Africa 7602, Email: kamper@sun.ac.za

Abstract – **Hub motors have always been considered as propulsion for electric vehicles, but not widely used due to various negative aspects. One of these is the uncertainty of the effect the added wheel mass has on the stability, safety and comfort of the vehicle. In this paper, frequency analysis as well as simulations of the system is done using a simple model that represents the vehicle suspension system and wheels. The results of the hub driven vehicle are compared to that of a standard vehicle. It is shown that the added wheel mass has no effect on the stability of the vehicle and that the frequency response is within the accepted comfort range.**

Index Terms – hub motor; natural frequency; suspension system;

I. INTRODUCTION

With the introduction of hub motors to the world of electric vehicles, a critical question has arisen: 'What effect has the added wheel mass of a hub motor on the safety and comfort of a vehicle?' Moving the propulsion from the vehicle body to the wheels can add up to 50 kg or more, per wheel, to the unsprung mass. Most of the research done on suspension systems has been done for standard vehicles [1]. No real investigations have been done on increased unsprung mass. A few recommendations state that the unsprung mass should not exceed 20% of the sprung mass [2]. Current road vehicles do not exceed this ratio and no real evidence supports this ratio.

By increasing the mass of the wheel, the wheel inertia is increased. Increased wheel inertia causes higher acceleration forces during road condition reaction. These forces put relatively high levels of stress on contact and connection points of the wheels. These forces can also cause degradation in ride comfort as experienced by the occupants of the vehicle.

The aim of the investigation is to study, through frequency analysis and simulation, the effect of moving mass from the sprung mass to the unsprung mass of a vehicle. The simulation results are compared with that of a standard vehicle to ascertain if it is possible to increase the unsprung mass of a vehicle.

The added mass has an effect on the handling of the vehicle as well. It is beyond the scope of the study to investigate this area, as the models and analyses are complex. These will be investigated in a later study to verify the first results as well as simulate vehicle handling. It is the opinion of the author that this study will give enough information to make sufficient conclusions on the effect of increased unsprung mass.

II. VEHICLE MODEL

A. Quarter Vehicle Suspension Model

The vehicle is modeled using a two-degree-of-freedom (2DOF) system [3]. The system comprises of two masses suspended by two sets of spring-damper systems. The advantage of using a 2DOF system is that it gives a simple yet accurate model of the vehicle's mass-suspension system and tire. The model allows observation of both suspension and tire deflection under applied road conditions. This model is a standard model used in suspension simulations [1, 4]. It represents a quarter of the vehicle and is thus called a quarter vehicle suspension model.

Fig. 1 represents the quarter vehicle suspension model. The masses, M_v and M_s, are the sprung and unsprung mass respectively. The two damper coefficients are given by *Bs* and *Bt* and the spring coefficients are given by K_s and K_t. The road, unsprung mass and sprung mass displacement is given by **x**, **y** and **z** respectively.

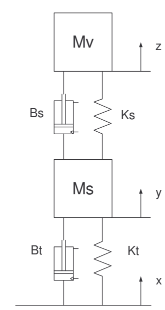

Fig. 1 Quarter vehicle suspension model.

As can be seen from Fig. 1, the displacement of the sprung mass, the unsprung mass and the road surface have different origins relative to each other. By using these frames of reference, the displacements, **y** and **z**, can refer to any solidly connected point on the unsprung or sprung mass.

B. Dynamic Equations

Newton's second law of motion is used to derive the dynamic equations of the system. Since there are two bodies, two equations need to be satisfied. These are:

$$M_V \ddot{z} = \sum F_V \tag{1}$$

$$M_S \ddot{z} = \sum F_S \tag{2}$$

The two forces F_v and F_s, which are the forces acting on the sprung and unsprung mass respectively can be found through inspection of Fig. 1. The dynamic equations of the system become:

$$M_V \ddot{z} = -K_S(z-y) - B_S(\dot{z}-\dot{y}) - M_V g \quad (3)$$

$$M_S \ddot{y} = K_S(z-y) + B_S(\dot{z}-\dot{y}) - K_T(y-x) - B_T(\dot{y}-\dot{x}) - M_S g \quad (4)$$

C. *Wheel Hop*

A very real phenomenon is that of the tire losing contact with the road surface, also known as "wheel hop". This phenomenon needs to be taken into account as it could happen during the simulations that the tire looses contact with the road due to either fast changing road conditions or the instability of the suspension system.

The last point of contact between the tire and the road surface occurs when the unsprung mass and the road surface is equally displaced from their respective origins i.e. **y-x=0**. The force due to the tire spring and damper are only exerted when the wheel is in contact with the road surface. Incorporating wheel hop into the dynamic equations, (4) becomes:

$$M_S \ddot{y} = K_S(z-y) + B_S(\dot{z}-\dot{y}) - K_T(y-x) - B_S(\dot{y}-\dot{x}) - M_S g \quad (5)$$
$$\text{if } (y-x) \le 0$$

$$M_S \ddot{y} = K_S(z-y) + B_S(\dot{z}-\dot{y}) - M_S g \quad (6)$$
$$\text{if } (y-x) \ge 0$$

The wheel hop phenomenon adds a non-linearity to the system. It has been decided that it can be neglected during the frequency analysis of the system, but not for the simulations. It will have little or no effect on the frequency response of the system. It adds complexity to the analysis with little increase in the accuracy of the results.

D. *Vehicle Parameters*

Two vehicles are compared in the study. One is a standard vehicle and the other a vehicle with a hub motor place in the rear wheels. The same total mass i.e. sprung and unsprung mass combined, is used for both vehicles. A total mass of 1500 kg was chosen. This is the mass of a fully laden vehicle (vehicle mass, passengers and payload). All constants used, such as damping and spring coefficients, are kept the same for both vehicles. Table I gives a list of all the constants used.

TABLE I
VEHICLE PARAMETERS

	Standard Vehicle		*Hub Driven Vehicle*	
	Total	*Model*	*Total*	*Model*
Total Mass (kg)	*1500*	*375*	*1500*	*375*
Sprung mass (kg)	*1340*	*335*	*1100*	*275*
Unsprung mass (kg)	*160*	*40*	*400*	*100*
Ks (N/m)	*36 000*	*36 000*	*36 000*	*36 000*
Bs (Ns/m)	*3000*	*3000*	*3000*	*3000*
Kt (N/m)	*110 000*	*110 000*	*110 000*	*110 000*
Bt (Ns/m)	*200*	*200*	*200*	*200*

The standard vehicle will serve as the control for the investigation and the hub driven vehicle as the experiment. As the simulation uses the quarter vehicle suspension model, all masses are a quarter of the real values.

III. FREQUENCY ANALYSIS

A. *Bode-plot Analysis*

It is important to verify that the suspension system and the vehicle are, through system frequency response analysis, stabile under changing road surface conditions. The simplest method to investigate the frequency response of the system is through a Bode-plot analysis. This can easily be done with the help of software like MatLab.

The transfer function is required to obtain the Bode-plot of the system. The transfer function can be mathematically derived from the dynamic equations or extracted from the linear model using MatLab. From MatLab the transfer function for the standard vehicle system is given as

$$G(s)_{ST} = \frac{8.527e-14s^3 + 4.093e-12s^2 + 2.463e4s + 2.955e5}{s^4 + 88.96s^3 + 3802s^2 + 2.516e4s + 2.955e5} \quad (7)$$

and the hub driven vehicle system as

$$G(s)_{HD} = \frac{6.395e-14s^3 + 3.638e-12s^2 + 1.2e4s + 1.44e5}{s^4 + 42.91s^3 + 1613s^2 + 1.226e4s + 1.44e5} \quad (8)$$

The transfer function can give an indication on the stability of the system. Both transfer functions have higher order poles than zeros. It can be seen that the second and third order zeros are small in comparison with the rest. These are good indicators that a system is stable. The Bode-plot will give an even better indication on the stability of the system.

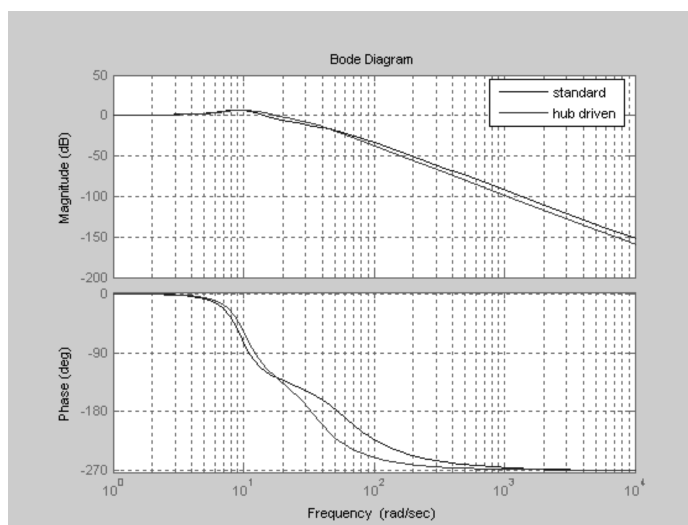

Fig. 2 Bode-plot of standard and hub driven vehicle.

A system is said to be unstable if it has a phase of -180 degrees at its crossover frequency. A system could also possibly be unstable if the magnitude is larger than 1 dB when the phase is equal to -180 degrees. From Fig. 2 and Table II it can be seen that the standard and hub driven vehicle do not meet these two criteria and thus are stable.

The dominant natural frequency of the system can easily be seen from the Bode-plot. This natural frequency is where the Bode-plot reaches a maximum. As both systems are 2DOF systems, two natural frequencies occur. The first or lower frequency will be the dominant natural frequency, with the higher second frequency being the damped natural frequency. The damped natural frequency is difficult to distinguish, but can be found by looking at the shape of the Bode-plot. The natural frequency can be calculated more accurately.

TABLE II
BODE-PLOT INFORMATION

	Standard	*Hub Driven*
First Natural Frequency (rad/s)	*9*	*10*
Second Natural Frequency (rad/s)	*60*	*40*
Crossover Frequency (rad/s)	*15*	*18*
-180 deg Frequency (rad/s)	*52*	*33*
Mag. at natural frequency (dB)	*7*	*7.5*

Something to note is that the two natural frequencies move closer together as the mass is shifted from the body to the wheels. When the natural frequencies are far apart the second is extremely damped and plays virtually no part in the oscillation of the system. As the two moves closer together, the second frequency starts playing a larger role. The two frequencies could move so close together, super positioning on each other, causing larger and unwanted oscillations.

B. *Natural Frequency Analysis*

The natural frequency of a system is the frequency at which a driving force causes maximum oscillation amplitude or even unbounded oscillation. In multiple-degree-of-freedom systems, the system has **n** number of natural frequencies. It is possible that the system resonates at all, some or none of its natural frequencies.

The natural frequencies of a 2DOF system are given as the square-root of its eigenvalues, that is

$$\omega_n = \sqrt{\lambda_n} \tag{9}$$

The eigenvalues of the systems are derived from the state space equations. The eigenvalues are given as:

$$\lambda_1 = \frac{1}{2}\left[\left(\frac{K_S + K_T}{M_S} + \frac{K_S}{M_V}\right) - \sqrt{\left(\frac{K_S + K_T}{M_S} + \frac{K_S}{M_V}\right)^2 - 4\left(\frac{K_S K_T}{M_S M_V}\right)}\right] \tag{10}$$

$$\lambda_2 = \frac{1}{2}\left[\left(\frac{K_S + K_T}{M_S} + \frac{K_S}{M_V}\right) + \sqrt{\left(\frac{K_S + K_T}{M_S} + \frac{K_S}{M_V}\right)^2 - 4\left(\frac{K_S K_T}{M_S M_V}\right)}\right] \tag{11}$$

The natural frequencies are calculated to be:

Standard vehicle: $\omega_1 = \sqrt{80.368} = 8.96$ rad/s or 1.43 Hz

$\omega_2 = \sqrt{3677.09} = 60.639$ rad/s or 9.65 Hz

Hub driven vehicle: $\omega_1 = \sqrt{96.349} = 9.82$ rad/s or 1.56 Hz

$\omega_2 = \sqrt{1494.56} = 38.659$ rad/s or 6.15 Hz

The calculated frequencies compare well with those given by the Bode-plot. The inaccuracy of the Bode-plot figures is due to the fact that the natural frequencies were obtained by inspection.

The human body is sensitive to certain frequency ranges [4]. The vehicle will be classified as uncomfortable if the first natural frequency falls within these ranges. It has been found that frequencies between 0.5 and 1 Hz cause a high occurrence of motion sickness. The human head and neck is especially sensitive to vibrations between 18 and 20 Hz. The abdomen region of the body is sensitive to vibrations between 5 and 7 Hz. Research has shown that a system with a natural frequency higher than 3 Hz is perceived as a "harsh ride". A ride is deemed to be comfortable near the 1.5 Hz mark.

Taking the above mentioned frequency regions into account; it is safe to stipulate a guideline stating that a comfortable system would have a dominant frequency between 1 and 3 Hz. It can be seen that the calculated frequencies fall within this ranges. Furthermore they are close to 1.5 Hz which is perceived as the optimum natural frequency.

C. *Payload Analysis*

The analysis in the previous section was done on the suspension system of a fully loaded standard and hub driven vehicle. The next step is to investigate the effect of varying the payload on the natural frequency of the system. The payload range from empty to fully load. A vehicle's curb weight is defined as the weight of the vehicle when it is fully operational plus one passenger. An electric vehicle's curb weight is generally less than that of a standard vehicle, as is the case in this section. The curb weights for a standard and hub driven vehicle is chosen as 900 kg and 750 kg respectively. Fig. 3 shows the dominant natural frequency for both the standard and hub driven vehicles for a range of payloads. The natural frequencies are calculated by means of the equations used in the previous section.

It can be seen that the varying payload has little effect on the natural frequency of the standard vehicle. On the other hand, the natural frequency of the hub driven vehicle shows significant variations due to the changing payload. This means that the hub driven vehicle will have a more varying ride response due to payload changes than the standard vehicle. However, both the vehicle's natural frequencies stay within the 1 to 3 Hz range, although the hub driven vehicle's frequency nears the 3 Hz limit when empty.

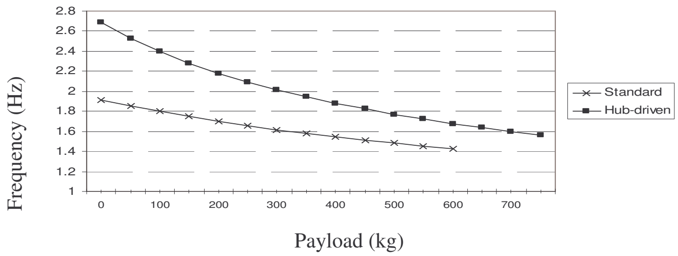

Fig. 3 Dominant natural frequency of standard and hub driven vehicles.

VI. Simulation

A. Simulation Model

The dynamic equations are implemented as a block diagram in MatLab /Simulink. This was done using standard Simulink blocks. All constants are imported into the model from a pre-created M-file. Fig. 4 gives the Simulink block diagram of the system. It can be seen that the wheel hop phenomenon was included in the model.

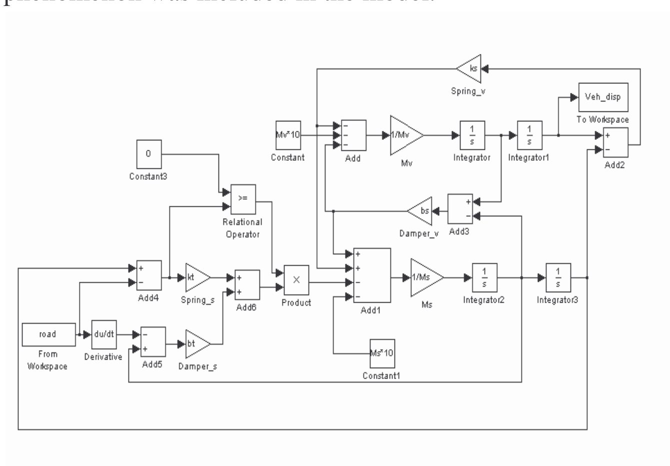
Fig. 4 Simulink model of mass-suspension system

B. Equilibrium Points

The simulation model of Fig. 4 takes static deflection of the suspension and tire into account. This is physically observed as suspension and tire sag. Both masses will thus have a negative displacement at equilibrium. Some models compensate for this by either adding pre-stress forces to the weight of the vehicle or removing the weight from the model. As the investigation is to determine what effect the changes in mass has on the system, no static deflection compensation should be done. With static deflection in mind, it is important to allow the simulation to reach equilibrium before any road input is given.

The static deflection points of the simulation were verified by comparing them with that of an actual vehicle. This is also done to verify the suspension constants used. The actual vehicle used has a mass of 1100 kg. The simulation parameters were changed to match these values. The simulation results of the static deflection points compare well with that of the actual vehicle.

B. Road Surface Input

Three types of inputs are used to investigate the suspension system's response to changing road surface condition. Again the hub driven vehicle is compared to a standard vehicle. The three road inputs used are a step input, a single bump and multiple or harmonic bumps. This is done at different vehicle speeds namely 5 and 50 km/h. Simulations are done at other speeds, but at these speeds two enough information is obtained. The vehicle's speed needs to be taken into account because of the fact that a faster vehicle has a shorter bump crossing time. The frequency components of the bump increases as the crossing time become shorter. Fig. 5 shows the crossing time at the simulated speeds.

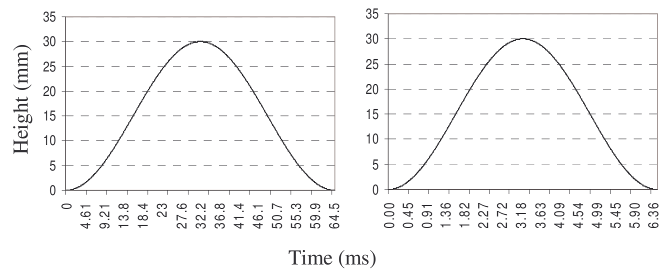

Fig. 5 Bump crossing at 5 and 50 km/h

C. Drop Test

The drop test is a standard test done on physical vehicles to measure suspension damping as well as oscillation frequencies. For simulation purposes the drop test can be done using a step input to the system. The standard step height is 0.08m [5]. In practical tests the vehicle is either be driven off a 0.08 m high ledge or dropped from a height of 0.08m.

From Fig. 6 it can be seen that the hub driven vehicle's sprung mass displacement is less negative than that of the standard vehicle. The suspension system exerts less force on the sprung mass due to the decreased weight. Less force means less suspension compression.

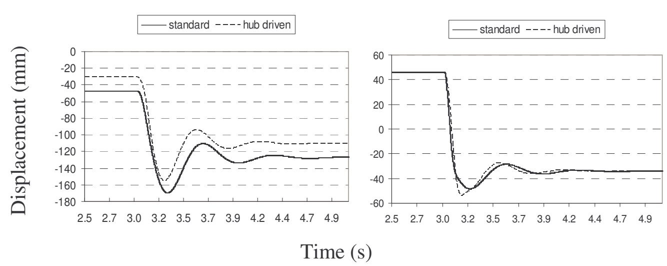

Fig. 6 Sprung and unsprung mass step response

No major differences were found from Fig. 6 in the displacement of the standard and hub driven vehicles' unsprung mass. The only occurrence worth noting is the peak of the first oscillation of the hub driven vehicle's unsprung mass displacement. This peak could compress the tire to such an extent, especially low profile tires, as to cause damage to the wheel rim.

D. Single and Multiple Bumps

The simulation model allows for any road surface data to be used as input to the system. This can be a single or a series of road bumps, either harmonic or irregular in shape. For the study, sinusoidal shaped bumps are used. These bumps are chosen to be 30 mm high and 90 mm wide.

The first simulation uses a single bump of the above mentioned dimensions as input. It was done for both the standard as well as the hub driven vehicle. Fig. 7 and 8 show the results of the displacement of the sprung and unsprung mass' displacement at different speeds.

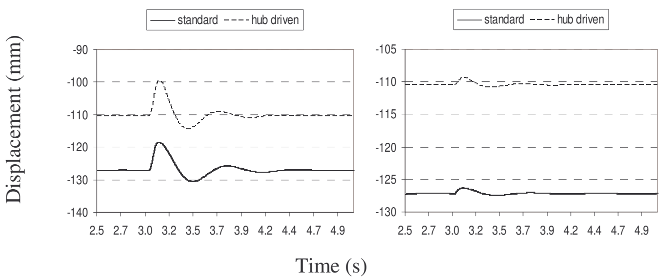

Fig. 7 Sprung mass displacement at 5 and 50 km/h (single bump).

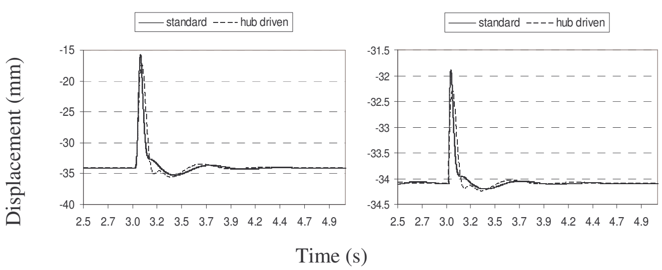

Fig. 8 Unsprung mass displacement at 5 and 50 km/h (single bump).

As can be seen, the systems are stable at the simulated speeds and no unwanted oscillations occur. As the speed increases, less of the bump is seen in the displacement of the sprung and unsprung mass. The increase in speed increases the frequency of the bump and thus moving further away from the natural frequency of the system. In reality this means that more of the bump is absorbed by the tire. When the hub driven vehicle is compared to that of the standard vehicle, no major differences are found in the displacement of the masses.

The next step is to investigate the systems response to multiple bumps and harmonic road surfaces, as seen in Fig. 10 and 11. These are implemented by using a series of the bumps used in the single bump simulation; the bumps are of the same dimensions.

The introduction of a series of bumps causes vibrations in the displacement of both the sprung and unsprung mass. The frequency and amplitude of these vibrations are important as they could cause discomfort to the occupants and possible damage to the suspension system and tire.

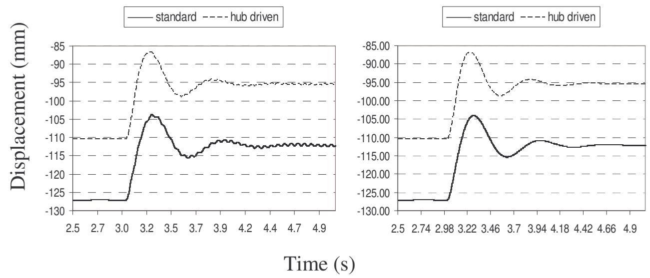

Fig. 9 Sprung mass displacement at 5 and 50 km/h (multi-bump).

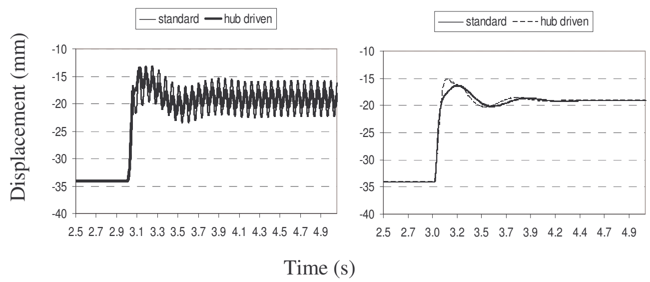

Fig. 10 Unsprung mass displacement at 5 and 50 km/h (multi-bump).

The displacement of sprung mass of both vehicles has noticeable vibrations at 5 km/h and decreases as the speed increases. The amplitude these vibrations experienced by the hub driven vehicle is found to be smaller than that of the standard vehicle. The frequencies are the same for the two vehicles as well.

The unsprung mass experiences more vibrations than the sprung mass. Again the amplitude is smaller for the hub driven vehicle than for the standard vehicle.

An interesting observation is that the system's response to the series of bumps starts to resemble that of a step response as the speed increases. At high speed the system seems to 'glide' across the bumps. This is due to the same fact mentioned for the single bump response. As the vehicle's speed increases, so does the input frequency increase and moves further away from the systems natural frequency. The vibrations are absorbed by the suspension system and the tire.

E. Wheel Hop

During all simulations the occurrence of the wheel hop phenomenon was monitored. It was found that it almost never occurs. The only simulation where it was found was during the drop test.

Studying the time it takes the wheel to regain contact with the road surface, it was found that the tire of the hub driven vehicle tire takes longer to return than that of the standard vehicle. This is due to the fact that for a smaller sprung mass, the suspension exerts a smaller force on the unsprung mass. This could cause weaker handling of the hub driven vehicle. On the other hand, the increased unsprung mass makes the hub driven vehicle's wheel less likely to leave the road surface. A more in-depth study is required to ascertain the full effect the added mass has on the handling of the vehicle.

F. *Force Analysis*

Standard vehicle wheels are rigid structures able to absorb high shock and vibration forces. By using hub motors a critical system is placed within the wheels of the vehicle. It is important to determine the magnitude of the forces exerted on the unsprung mass. If these forces are too high and are transferred through the motor, it could lead to quicker wearing of components or even damage to the motor. The forces are calculated for both single and multiple bump road inputs. Table III and IV gives the maximum forces for these cases.

TABLE III
MAXIMUM FORCE EXERTED ON UNSPRUNG MASS (SINGLE BUMP)

Speed	*Standard*	*Hub Driven*
5 km/h	*1215 N*	*1984 N*
50 km/h	*4525 N*	*4800 N*
100 km/h	*7475 N*	*7680 N*

TABLE III
MAXIMUM FORCE EXERTED ON UNSPRUNG MASS (MULTI BUMP)

Speed	*Standard*			*Hub driven*		
	Initial	*Mag.*	*Freq.*	*Initial*	*Mag.*	*Freq.*
5 km/h	*1219N*	*1550N*	*15.3 Hz*	*1970N*	*1775 N*	*14.3 Hz*
50 km/h	*4630N*	*3450N*	*166.7Hz*	*5880N*	*3460N*	*166.7Hz*
100 km/h	*8860N*	*6250N*	*333.3Hz*	*8830N*	*6200N*	*333.3 Hz*

The unsprung mass receives an initial shock force when the tire hits the first of the bumps. This initial force has the same magnitude as for the case of a single bump. After the initial force, the unsprung mass experiences the vibrations caused by traveling across the series of bumps. Although the oscillations in displacement decrease with an increase in speed, it can be seen that the force increases with the increase in speed.

At this stage it can not be decided whether the results of the force analysis is within limits. The force exerted on the unsprung mass of the hub driven vehicle is not remarkably higher than that experienced by the standard vehicle. The only way to determine whether or not these results are acceptable is to investigate the physical structure of the hub motor. Finite element strength analysis can determine if the wheel structure and motor can withstand the shock and vibration forces.

VI. FUTURE WORK

The work described in this paper is the first step in studying the effect of the added mass of a hub motor on the response of a vehicle's suspension system. As mentioned in a previous section, the effect of vibrations on the structural integrity of the hub motor can not be found from the system simulation results. A full strength analysis is to be done through the use of finite element software.

It is important to verify the simulation results with the use of practical experiments. An experimental test setup has been devised where a vehicle is modified by adding mass to its wheels. This is done by attaching a weight to the axel of the vehicle. This weight represents the added hub motor. The vehicle can be driven over different road surfaces at different speeds. The vehicle is still powered by an internal combustion engine. Measurement can be taken and even handling test can be done before any hub motor is attached. Fig. 11 shows the weight attached to the vehicle's axel and the rim assembly.

Fig. 11 Axel weight representing the hub motor mass.

V. CONCLUSIONS

The simulation results show that the displacement of the sprung and unsprung mass of the hub driven vehicle does not differ much from that of the standard vehicle. The vibrations experienced by the sprung mass and thus by the occupants of the vehicle does not decrease the comfort of the vehicle with the addition of the hub motor.

Natural frequency calculations show that the natural frequency for the hub driven vehicle fall within the acceptable frequency ranges of driver comfort and safety. The hub driven vehicle shows increased variation in natural frequency caused by payload variations. The study has shown that the suspension system of a standard vehicle can be used for a hub driven vehicle without loss of comfort and safety. It is possible to improve the comfort of the vehicle by designing the suspension system to match the mass distribution.

It is the opinion of the authors that hub motors can be used successfully as propulsion for electric or hybrid electric vehicles.

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